

A Statistical MIMO FSO Channel Model for the Analysis of Outage Probability versus Signal-to-Noise Ratio

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ABSTRACT

A novel statistical channel model for multiple-input multiple-output (MIMO) free-space optical (FSO) communication systems impaired by atmospheric and misalignment fading is developed. A slow-fading channel model is considered and the outage probability is derived as a performance measure. The diversity gain defined as the signal-to-noise ratio (SNR) exponent at high SNR is analyzed. Interestingly in the presence of misalignment fading the diversity gain depends only on the misalignment variance and is independent of the number of transmitters M and receivers N . Increasing the number of transmitters and receivers only results in a lower probability of outage for given SNR, however, the rate of change is unaffected. Contrary to this, the diversity gain of MIMO FSO systems in the presence of atmospheric fading and no misalignment is shown to be proportional to the number of transmitters and receivers, particularly the product M and N .

Keywords: LS, FSO, MIMO, SNR.

I. INTRODUCTION

The use of multiple antennas at the transmitter and receiver in wireless systems, popularly known as MIMO (multiple-input multiple-output technology, has rapidly gained in popularity over the past decade due to its powerful performance-enhancing capabilities. Communication in wireless channels is impaired predominantly by multi-path fading. Multi-path is the arrival of the transmitted signal at an intended receiver through differing angles and differing time delays and differing frequency (i.e., Doppler) shifts due to the scattering of electromagnetic waves in the environment. Consequently, the received signal power fluctuates in space (due to angle spread) and/or frequency (due to delay spread) and/or time (due to Doppler spread) through the random superposition of the impinging multi-path components. This random fluctuation in signal level, known as fading.

MIMO technology constitutes a breakthrough in wireless communication system design. The technology offers a number of benefits that help meet the challenges posed by both the impairments in the wireless channel as well as resource constraints. In addition to the time and frequency

dimensions that are exploited in conventional single-antenna (single-input single-output) wireless systems, the leverages of MIMO are realized by exploiting the spatial dimension (provided by the multiple antennas at the transmitter and the receiver).

The advantages of multiple-input multiple-output (MIMO) systems have been widely acknowledged, to the extent that certain transmit diversity methods (i.e., Alamouti signaling) have been incorporated into wireless standards. Although transmit diversity is clearly advantageous on a cellular base station, it may not be practical for other scenarios. Specifically, due to size, cost, or hardware limitations, a wireless agent may not be able to support multiple transmit antennas.

1.1 BENEFITS OF MIMO TECHNOLOGY

The benefits of MIMO technology that help achieve such significant performance gains are array gain, spatial diversity gain, spatial multiplexing gain and interference reduction. These gains are described in brief below.

➤ Array gain

Array gain is the increase in receive SNR that results from a coherent combining effect of the wireless signals at a receiver. The coherent combining may be realized through spatial processing at the receive antenna array and/or spatial pre-processing at the transmit antenna array. Array gain improves resistance to noise, thereby improving the coverage and the range of a wireless network.

➤ Spatial diversity gain

As mentioned earlier, the signal level at a receiver in a wireless system fluctuates or fades. Spatial diversity gain mitigates fading and is realized by providing the receiver with multiple (ideally independent) copies of the transmitted signal in space, frequency or time. With an increasing number of independent copies (the number of copies is often referred to as the diversity order), the probability that at least one of the copies is not experiencing a deep fade increases, thereby improving the quality and reliability of reception.

➤ Spatial multiplexing gain

MIMO systems offer a linear increase in data rate through spatial multiplexing, i.e., transmitting multiple, independent data streams within the bandwidth of operation. Under suitable channel conditions, such as rich scattering in the environment, the receiver can separate the data streams.

➤ Interference reduction and avoidance

Interference in wireless networks results from multiple users sharing time and frequency resources. Interference may be mitigated in MIMO systems by exploiting the spatial dimension to increase the separation between users. For instance, in the presence of interference, array gain increases the tolerance to noise as well as the interference power, hence improving the signal-to-noise-plus-interference ratio (SINR). Additionally, the spatial dimension may be leveraged for the purposes of interference avoidance, i.e., directing signal energy towards the intended user and minimizing interference to other users. Interference reduction and avoidance improve the coverage and range of a wireless network. In general, it may not be possible to exploit simultaneously all the benefits described above due to conflicting demands on the spatial degrees of freedom.

1.2 MIMO IN CELLULAR NETWORKS

In a cellular wireless communication network, multiple users may communicate at the same time and (or) frequency. The more aggressive the reuse of time and frequency resources, the higher the network capacity will be, provided that transmitted signals can be detected reliably. Multiple users may be separated in time (time-division) or frequency (frequency-division) or code (code-division). The spatial dimension in MIMO channels provides an extra dimension to separate users, allowing more aggressive reuse of time and frequency resources, thereby increasing the network capacity.

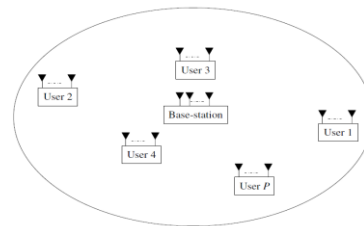


Fig1. MIMO cellular system. A base-station with L antennas communicates with P users, each equipped with M antennas.

Fig.1 is the schematic of a cell in a MIMO cellular network. A base-station equipped with L antennas communicates with P users, each equipped with M antennas. The channel from the base-station to the users (the downlink) is a broadcast channel (BC) while the channel from the users to the base-station (the uplink) is a multiple-access channel (MAC). The set of rate-tuples (R_1, R_2, \dots, R_P) that can be reliably supported on the downlink or uplink constitutes the capacity rate region for that link. Recently, an important duality has been discovered between the rate regions for the downlink and uplink

Channels differences between users. However, with rich scattering and $L \geq PM$, we can expect that the spatial signatures of the users are well separated to allow reliable detection. Using a multi-user ZF receiver will allow perfect separation of all the data streams at the base-station, yielding a multi-user multiplexing gain of PM. The use of more complex receivers for multi-user detection and the associated performance trade-offs. A similar thought experiment can be applied for the downlink, where the base-station exploits the spatial dimension to beam information intended for a particular user towards that user and steers nulls in the directions of the other users, thus completely eliminating interference.

1.3 DISTRIBUTED MIMO

While MIMO technology provides substantial performance gains, the cost of deploying multiple antennas at terminals in a network can be prohibitive, at least for the immediate future. Distributed MIMO is a means of realizing the gains of MIMO with single-antenna terminals in a network, allowing a gradual migration to a true MIMO network. The approach requires some level of cooperation between network terminals.

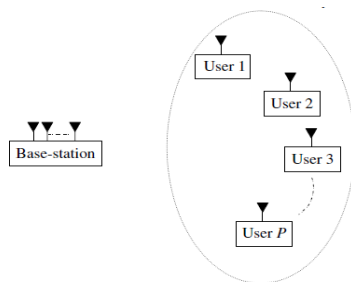


Fig.2. Distributed MIMO: multiple users cooperate to form a virtual antenna array that realizes the gains of MIMO in a distributed fashion

II. EXISTING SYSTEM

The effects of atmospheric and misalignment fading are analyzed in the case of single input single-output (SISO) FSO channels. Atmospheric turbulence fades the signal intensity at the receiver giving rise to turbulence-induced scintillation. The outage capacity of SISO is analyzed by effects of atmospheric turbulence and misalignment.

DISADVANTAGES

- Diversity Gain is reduced
- It is used for single input and single output i.e. one transmit and one receive antenna.

III. PROPOSED SYSTEM

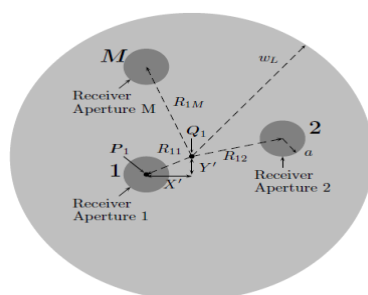


Fig. 3. Detector and beam footprint for transmitter #1 with misalignment at the detector plane.

Fig. 3 shows the block diagram of the system. A statistical model for MIMO FSO channels where both atmospheric and misalignment fading are considered. The outage capacity is analyzed by considering an intensity-modulated laser with pulse-amplitude-modulation (PAM). Diversity gain is analyzed using a log-normal distribution for the atmospheric fading. The outage probabilities are derived by taking account of different misalignment fading.

A) TECHNIQUE

Free space optics Free-space optical communication (FSO) is an optical communication technology that uses light propagating in free space to transmit data for telecommunications or computer networking. "Free space" means air, outer space, vacuum, or something similar. This contrasts with using solids such as optical fiber cable or an optical transmission line. The technology is useful where the physical connections are impractical due to high costs or other considerations.

Free-space point-to-point optical links can be implemented using infrared laser light, although low-data-rate communication over short distances is possible using LEDs. Infrared Data Association (IrDA) technology is a very simple form of free-space optical communications. Free Space Optics is additionally used for communications between spacecraft. Maximum range for terrestrial links is in the order of 2 to 3 km (1.2 to 1.9 mi), but the stability and quality of the link is highly dependent on atmospheric factors such as rain, fog, dust and heat. Amateur radio operators have achieved significantly farther distances using incoherent sources of light from high-intensity LEDs.

MIMO: Multiple-input and multiple-output (MIMO) is the use of multiple antennas at both the transmitter and receiver to improve communication performance. It is one of several forms of smart antenna technology. It offers significant increases in data throughput and link range without additional bandwidth or increased transmit power.

B) MODULES NAME

Channel Model

- Atmospheric and Misalignment Fading Statistical MIMO FSO Channel Model

- Statistical Channel Model
- Probability Of Outage And Diversity Gain
- Asymptotic Channel Capacity at High SNR
 - Probability of Outage with Misalignment

Diversity Gain of MIMO FSO Channels

- Diversity Gain of MIMO FSO Channels
- Unidirectional Misalignment
- No Misalignment

C) Module Explanation: Channel Models

The model considers MIMO FSO system with M transmitters (lasers) and N receivers (apertures). In all cases, intensity modulated PAM signaling with direct detection is considered.

The received $N \times 1$ vector $y = [y_1, \dots, y_N]^T$ is given by

$$y = H^T x + z$$

where H is an $M \times N$ channel matrix where the entry $H_{mn} \geq 0$ represents the channel gain from transmitter m to receiver n with $m=1, \dots, M$ and $n=1, \dots, N$ and $(\cdot)^T$ is the transpose operator. The vector $x = [x_1, \dots, x_M]^T$ is the transmitted set of symbols and $z = [z_1, \dots, z_N]^T$ is a noise vector of independent components modeled as signal independent white and Gaussian distributed.

The signal-to-noise ratio is defined as $SNR = P/\sigma$ and the channel gain

$$H = \sum_{n=1}^N \sum_{m=1}^N H_{mn}$$

accounts for the combined effects of atmosphere and misalignment fading where

$$H_{mn} = H_{mn}^a H_{mn}^p$$

where H_{mn}^a and H_{mn}^p are independent random variables representing the atmospheric and time varying misalignment (pointing) fading respectively between transmitter m and receiver n.

In the weak turbulence regime the channel gain due to atmospheric turbulence is well modeled by

$$H_{mn}^a = e^{X_{mn}}$$

where X_{mn} is a Gaussian random variable. Assume that all X_{mn} are modeled as independent and identically distributed (i.i.d) random variables.

For a radial displacement of R_{mn} in the receiver plane between the center of transmitter beam m and the center of aperture n, the loss due to misalignment is

$$H_{mn}^p \approx A_0 e^{-2R_{mn}^2/w^2}$$

where A_0 is the equivalent receiver area and w is the equivalent beam waist at receiver.

IV. STATISTICAL MIMO FSO CHANNEL MODEL

Misalignment fading depends on the geometric arrangement of the transmit lasers and receive apertures. In this work, we consider $M = N$. The spacing between transmit lasers is d which is also the spacing between receiver apertures. Note that d is assumed to be larger than the coherence length of atmospheric fading. We further assume that initially each transmit laser is aligned to the corresponding receive aperture. Let \mathcal{P} denote the set of coordinates of all receiver apertures. Also, let $P_n \in \mathcal{P}$ denote the coordinate vector of the n^{th} receiver and Q_m denote the coordinate vectors of the center of the m^{th} transmit beam footprint at the receiver plane. Thus, the beam footprint at the receiver for a random displacement of X' in the x-direction and Y' in the y-direction is

$$Q_m = \begin{bmatrix} X' \\ Y' \end{bmatrix} + P_m$$

for the case of no misalignment $P_m = Q_m$.

Statistical Channel Model

The channel gain between transmitter m and receiver n is given by

$$H_{mn} = H_{mn}^a H_{mn}^p = A_0 e^{X_{mn} - 2R_{mn}^2/w^2}$$

The total channel gain is given by

$$H = A_0 e^{-T} \sum_{n=1}^N \sum_{m=1}^N e^{X_{mn} - U_{mn}}$$

D) PROBABILITY OF OUTAGE AND DIVERSITY GAIN

Since the coherence time of the FSO channel fading is much larger than the symbol duration, a slow-fading model is considered in the analysis. The outage probability is used as a performance metric and the diversity gain for $M = N$ transmitters and receivers is computed for a variety of spatial arrangements and misalignments.

Asymptotic Channel Capacity at High SNR

Consider a PAM modulated IM/DD FSO communication system subject to amplitude non-negativity and average constraints. Although a closed form is not known, at high SNR the asymptotic channel capacity can be efficiently bounded. The capacity of FSO channel is upper bounded as

$$C(SNR) \leq \log_2 \left(\sqrt{\frac{e}{2\pi}} SNR + 2 \right)$$

Considering an arbitrary input distribution in the mutual information expression results in a capacity lower bound. A lower bound on $C(\text{SNR})$, (bits/channel use), can be developed in terms of mutual information I and entropy H as

$$C(\text{SNR}) \geq I(X; Y | H = h)_{f_X(x) \triangleq \text{exp}}$$

$$C(\text{SNR}) \approx \log_2 \left(\sqrt{\frac{e}{2\pi}} \text{SNR} \right)$$

Probability of Outage with Misalignment

For a given transmission rate, there is a finite probability that the transmitted rate exceeds the instantaneous mutual information of the channel. This probability is denoted as the outage probability and at a given rate R_0 is defined as

$$P_{\text{out}}(R_0) = \text{Prob}(C < R_0)$$

Consider the optical channel corrupted by atmospheric and misalignment fading, the outage probability at high SNR is given by

$$C(\text{SNR}) \leq \log_2 \left(\sqrt{\frac{e}{2\pi}} \text{SNR} + 2 \right)$$

$$P_{\text{out}}(R_0) = \text{Prob}_{f_H(h)} \left[\log_2 \left(\sqrt{\frac{e}{2\pi}} \text{SNR} \right) < R_0 \right]$$

V. SIMULATION RESULTS

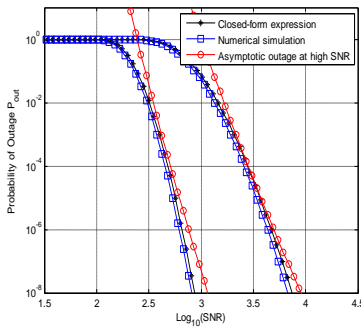


Fig. 4. Probability of outage versus SNR for 2×2 and 4×4 MIMO FSO systems arranged as $\mathcal{P}2 \times 2$ and $\mathcal{P}4 \times 4$ resp. with unidirectional misalignment fading.

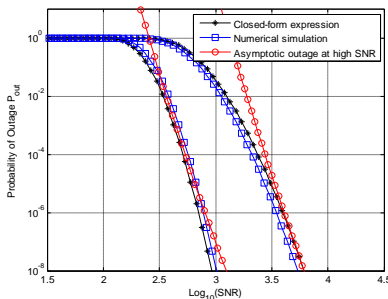


Fig. 5. Closed-form expression of the probability of outage versus SNR for 2×2 and 4×4 MIMO FSO systems with different atmospheric fading parameter.

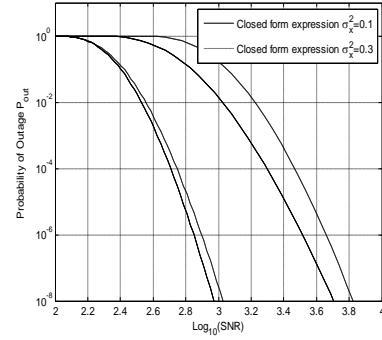


Fig.6 Probability of outage versus SNR for 4×4 MIMO FSO system $\sigma_s^2=0.3, 0.1$ and 0.01 .

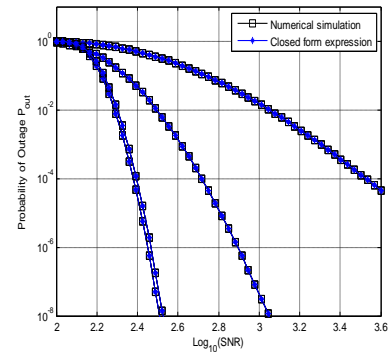


Fig.7 Probability of outage versus SNR for 2×2 and 4×4 MIMO FSO systems arranged as $\mathcal{P}2 \times 2$ and $\mathcal{P}4 \times 4$ respectively with symmetric misalignment fading.

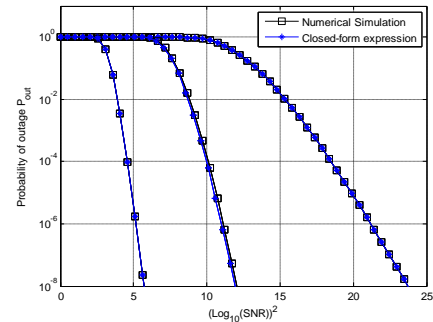


Fig. 8. Probability of outage versus SNR for 1×1 SISO, 2×2 , 4×4 and 6×6 MIMO FSO systems.

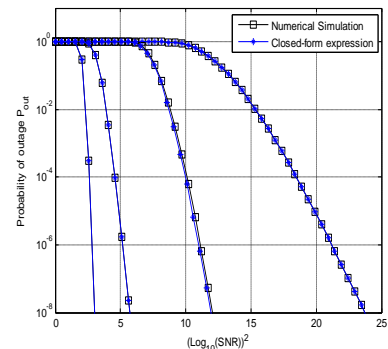


Fig. 9. Probability of outage versus SNR for 1×1 SISO, 2×2 , and 4×4 MIMO FSO systems.

Advantages of Proposed System

- Increasing the diversity gain
- Misalignment fading is reduced

VI. CONCLUSIONS

A novel generalized statistical model for MIMO FSO channels impaired by atmospheric and misalignment fading is developed. The derived model is utilized to study the outage probability of FSO channels and the diversity gain at high signal-to-noise ratio. Closed-form expressions for the outage probability are derived taking into account different misalignment fading scenarios.

It is shown that, in the presence of atmospheric and misalignment fading, the diversity gain depends only on the misalignment parameters and is independent of both the number of transceivers and atmospheric fading parameters. Contrarily, when atmospheric fading is the only channel impairment, i.e., no misalignment fading, the diversity gain depends on the number of transceivers. Thus a larger diversity gain can be achieved by increasing the number of transmitters and receivers. In all cases increasing the number of transmitters and receivers decreases the outage probability for a given SNR. However, in order to have independent channels gains it is required to sufficiently increase the spacing between receivers which is often practically difficult.

An alternative approach is to utilize a large single-aperture with the equivalent area of the N apertures. This approach provides simple system structure and reduces the fading variance via aperture averaging. Note that, in this case the fading is correlated across the aperture and hence its variance is larger than that of a system with multiple apertures and independent fading at each aperture.

REFERENCES

- [1] J. M. Kahn and J. R. Barry, "Wireless infrared communications," *Proc. IEEE*, vol. 85, pp. 265–298, Feb. 1997.
- [2] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*, 1st edition. Cambridge University Press, 2005.
- [3] M. Razavi and J. H. Shapiro, "Wireless optical communications via diversity reception and optical preamplification," *IEEE Trans. Wireless Commun.*, vol. 4, pp. 975–983, May 2005.
- [4] X. Zhu and J. Kahn, "Free space optical communication through atmospheric turbulence channels," *IEEE Trans. Commun.*, vol. 50, pp.1293–1300, Aug. 2002.
- [5] S. M. Navidpour, M. Uysal, and M. Kavehrad, "BER performance of free-space optical transmission with spatial diversity," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 2813–2819, Aug. 2007.
- [6] T. A. Tsiftsis, H. G. Sandalidis, G. K. Karagiannidis, and M. Uysal, "Optical wireless links with spatial diversity over strong atmospheric turbulence channels," *IEEE Trans. Wireless Commun.*, vol. 8, pp. 951– 957, Feb. 2009.
- [7] N. Letzepis and A. G. i Fábregas, "Outage probability of the Gaussian MIMO free-space optical channel with PPM," *IEEE Trans. Commun.*, vol. 57, pp. 3682–3690, Dec. 2009.
- [8] "Outage probability of the free-space optical channel with doubly stochastic scintillation," *IEEE Trans. Commun.*, vol. 57, pp. 2899–2902, Oct. 2009.
- [9] S. M. Haas and J. H. Shapiro, "Capacity of wireless optical communications," *IEEE J. Sel. Areas Commun.*, vol. 21, pp. 1346–1356, Oct. 2003.
- [10] S. G. Wilson, M. Brandt-Pearce, Q. Cao, and J. H. Leveque, "Free-space optical MIMO transmission with Q-ary PPM," *IEEE Trans. Commun.*, vol. 53, pp. 1402–1412, Aug. 2005.

BIOGRAPHY



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